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RESEARCH INTO METHODS OF DETERMINING THE RELATIVE HEIGHTS OF PHYSIOGRAPHIC FEATURES OF THE MOON.

by

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The research carried out with the support of this contract during the first year of its existence has been concerned with the following problems:

(a) re-determination of the libration constants of the Moon;

(b) evaluation and measurements of the old Paris photographic plate of the Moon - covering the full range of libration - for determining the departures of the actual surface of the Moon from a sphere; and

(c) development of methods for the determination of small differences in height (or depth) of very shallow surface features - such as the lunar domes, rills, and wrinkle ridges - above (or below) the surrounding landscape.

Distinct contributions to all three of these lines of research have been made in the course of the year; and the progress made along each will be discussed below in turn. The work under (a) was performed by Professor K. Koziel of the University of Krakow, President of the I.A.U. Commission No. 17 (Selenodesy), who visited Manchester for a period of six months (October 1961 through March 1962) largely to use the

University's electronic computer for this purpose. In this work Professor Koziel was joined by his Krakow associates, J. Mietelski (October-December 1961) and J. Maslowski (January-March 1962), ably assisted at the electronic computer by Mrs. Mary Gorman of our own staff.

Preparations for work on task (b) have been made in collaboration with Dr. Th. Weimer, of the Paris Observatory, to modernize first the equipment available to him at Paris for the purpose of the contemplated research; while work under (c) was pursued throughout the year at Manchester by Messrs D. Dale and T. W. Rackham, under the supervision of Professor Z. Kopal. The contributions of Mr. T. W. Rackham to three-dimensional topography of the Moon earned him the Master's degree at the University of Manchester in December 1961; and it is expected that Mr. Dale's efforts in this field will earn him the same academic reward during the coming session.

(a) Re-determination of the Libration Constants of the Moon.

The importance of determining the exact shape of the Moon, and of the parameters (such as the difference between the principal moments of inertia) characterizing the internal structure of the lunar globe can hardly be overestimated at the present time, when plans are under way to land men on the Moon before the end of the present decade. But the first astronauts landing on the surface of our satellite will need good charts of the Moon, similar to the charts of the Earth; and the photographs - even the best ones - cannot replace such charts. The basic data for the construction of these charts must come from selenodetic work - similar in method to terrestrial geodesy, except for the fact that all measurements must be carried out, by means of suitable telescopes, at a distance which never becomes less than 356,000 km. The tasks of selenodesy are further complicated by the fact that, on the Moon, we do not possess any obvious surface of reference corresponding to the terrestrial sea-level. In fact, the only way to establish the deviations of the actual surface of the Moon from a sphere is from the stereoscopic effects exhibited by such deformations in the course of lunar librations.

The observations of apparent motions of specific points of the lunar surface with respect to the limb of the Moon in the course of a full libration cycle have been one of the principal tasks of selenodesy in the past century; and, as is well known, the most important instrument designed specifically for this purpose has been Bessel's heliometer. In heliometric measurements - which constitute so far the vast majority of the libration measurements of the Moon - it has been customary to refer the position of the crater Mösting A, situated near the centre of the Moon's apparent disk (Fig.1.) to the illuminated limb of the Moon. This is done by measuring the distance s_o (s observatum) of this crater from the limb in specified position angles p, in order to determine the position of Mösting A as referred to the centre of mass of the lunar ellipsoid.

From Bessel's time until recently, the adjustment of the observations of the Moon's libration was done in two stages. First, the auxiliary unknowns of the problem, (i.e., the corrections to the rectangular plane co-ordinates of the crater Mösting A) were found, and only the second adjustment gave the proper unknowns of the problem - i.e., the corrections to the selenographic co-ordinates λ_0, β_0 as well as to the Moon's radius to the crater Mösting A (so-called h) and also the corrections to the mean inclination of the lunar equator to the ecliptic $\mathcal{J}$ and to the mechanical ellipticity of the Moon $f = \frac{B(C - B)}{A(C - A)}$, where A, B and C denote the principal moments of inertia. From the very beginning this manner of treatment created great difficulties in the choice of weights of the right-hand sides of the observation equations in the second adjustment. These right-hand sides are not independent, as

Figure 1.

Central part of the apparent disk of the Moon, including the craters Ptolemy (centre), Alphonsus, (top) Flammarion (bottom) and Mösting A (indicated by arrow).

they were obtained from the same equations in the first stage. But the adjustment of such equations in the second stage is incorrect from the point of view of the least-squares method, and no choice of weights can secure here a correct solution. The reason for such a state of affairs was the difficulty in the execution of the very extended calculations.

In 1948 Professor Koziel published a paper about the Moon's libration in which he reported some progress with this problem: but in 1956 he published for the first time the results for a special case; and in 1961 he worked out in Manchester a general solution of the problem, correct from the point of view of the least-squares method. The correct mathematical form of the final observation equations becomes:

$$\begin{Bmatrix} d\xi \\ d\eta \\ d\zeta \\ d\,\text{[illegible]} \\ df_1 \\ df_2 \\ u \\ w \\ dR_0 \end{Bmatrix} \left[\sec \pi'' \begin{Bmatrix} -\sin(p-\pi'') \\ -\cos(p-\pi'') \\ 1 \end{Bmatrix} \begin{Bmatrix} A & B & C & D & \mathcal{E}_1 & \mathcal{E}_2 & F & G & 0 \\ A' & B' & C' & D' & \mathcal{E}_1' & \mathcal{E}_2' & F' & G' & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{R'_{\mathbb{C}}}{R_0} \end{Bmatrix} \right] = \{ S_0 - S_c$$

The differential co-efficients A, A^I, B, ... G, G^I occurring in these equations can be found from the formulae obtained on the basis of differential formulae of spherical polygonometry. The adjustment of the libration observations whose left-hand sides contain the fundamental unknowns of the problem and, on the right-hand sides, the observed residuals $(s_o - s_c)$, permits the adjustment to be carried out in accordance with the principles of the least-squares method. Moreover, the present form of the observation equations enables use to be made of even the so-called quite incomplete observation evenings - i.e., such evenings as, in the extreme case, consist of only one measurement of the illuminated limb of the Moon's disc. A further advantage of observation equations of this type is that they allow the introduction, into their right-hand sides, of the corrections to s_o for the irregularities of the Moon's limb or any other corrections and then a parallel adjustment may be carried out without any change to the left-hand sides.

It is interesting that the simplest and most rigorous mathematical form of the observation equations as given above, is also the simplest for the electronic computer "Mercury". A total of 3328 observations taken during 340 evenings in the period 1877-1915 have been used. Of these, 46 observations were rejected, as the s_o - s_c's were four times larger than the

mean error of one observation equation. However, in no case did a whole evening have to be rejected; since, if we use the type of observation equation under consideration, it is possible to utilise every observation. When Professor Koziel came to Manchester he had already prepared the differences $s_o - s_c$ for the following series of observations:

Strasbourg Series (1877-1879) by K. Koziel;
Dorpat Series (1884-1885) by K. Koziel;
Bamberg Series (1890-1912) by J. Maslowski; and
Kazan Series (1910-1915) by J. Mietelski.

The $s_o - s_c$ were calculated by using the formulae:

$$\begin{Bmatrix} \cos K \\ \sin K \cos \bar{\pi} \\ \sin K \sin \bar{\pi} \end{Bmatrix} = \begin{Bmatrix} \sin \beta \\ \cos \beta \end{Bmatrix} \begin{Bmatrix} 1 & 0 & 0 \\ 0 & -\sin(\Omega - l_{☾} - \lambda) & -\cos(\Omega - l_{☾} - \lambda) \end{Bmatrix} \begin{Bmatrix} \cos I & \sin I & 0 \\ -\sin I & \cos I & 0 \\ 0 & 0 & 1 \end{Bmatrix}$$

$$\times \begin{Bmatrix} 1 & 0 & 0 \\ 0 & \cos \Omega & \sin \Omega \\ 0 & -\sin \Omega & \cos \Omega \end{Bmatrix} \begin{Bmatrix} \cos E & \sin E & 0 \\ -\sin E & \cos E & 0 \\ 0 & 0 & 1 \end{Bmatrix} \begin{Bmatrix} 1 & 0 & 0 \\ 0 & -\sin \alpha'_{☾} & -\cos \alpha'_{☾} \\ 0 & \cos \alpha'_{☾} & -\sin \alpha'_{☾} \end{Bmatrix} \begin{Bmatrix} -\sin \delta'_{☾} & \cos \delta'_{☾} & 0 \\ -\cos \delta'_{☾} & -\sin \delta'_{☾} & 0 \\ 0 & 0 & 1 \end{Bmatrix},$$

which represent the solution of a spherical pentagon on the selenographic sphere.

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which represent the solution of a spherical pentagon on the selenographic sphere.

Then,

$$\tan \bar{\sigma} = \frac{\sin h' \sin K}{1 - \sin h' \cos K} ,$$

where h' is the topocentric radius for Mösting A. After computing π'' from the formula

$$\sin \pi'' = \frac{\bar{\sigma}}{R'_{\mathbb{C}}} \sin(p - \bar{\pi})$$

we finally have,

$$s_c = \left[R'_{\mathbb{C}} - \bar{\sigma} \cos(p - \bar{\pi} - \pi'') \right] \sec \pi''$$

and from observations we have s_o.

Turning now to the adjustments and starting from Hayn's value of the libration constants, Professor Koziel has divided the adjustment calculations into seven parts.

1. Professor Koziel gave to Mrs. Gorman, for programming for the "Mercury" computer, the following formulae which have recently been published in English for the first time in Physics and Astronomy of the Moon, ed. Z. Kopal (Academic Press, London 1962):

$$\begin{Bmatrix} A & A' \\ B & B' \\ C & C' \end{Bmatrix} = \frac{R'_{\mathbb{C}}}{\Delta R_o} \begin{Bmatrix} A_{12} & A_{13} \\ A_{22} & A_{23} \\ A_{32} & A_{33} \end{Bmatrix} ,$$

$$\begin{Bmatrix} D & \mathcal{E} \\ D' & \mathcal{E}' \end{Bmatrix} = \frac{1}{\Delta} \begin{Bmatrix} \bar{a} & \bar{a}' \\ \bar{b} & \bar{b}' \\ \bar{c} & \bar{c}' \end{Bmatrix} \begin{Bmatrix} A_{12} & A_{13} \\ A_{22} & A_{23} \\ A_{32} & A_{33} \end{Bmatrix},$$

$$\begin{Bmatrix} \bar{a} \\ \bar{b} \\ \bar{c} \end{Bmatrix} = h' \begin{Bmatrix} -\cos\beta\cos\lambda \\ -\cos\beta\sin\lambda \\ \sin\beta \end{Bmatrix} \begin{Bmatrix} 0 & 0 & -\sin(l_e - \Omega) \\ 0 & 0 & -\cos(l_e - \Omega) \\ \sin(l_e - \Omega) & \cos(l_e - \Omega) & 0 \end{Bmatrix},$$

$$\begin{Bmatrix} \bar{a}' \\ \bar{b}' \\ \bar{c}' \end{Bmatrix} = h' \begin{Bmatrix} \frac{d\lambda}{df} \\ \frac{d\beta}{df} \end{Bmatrix} \begin{Bmatrix} \cos\beta\sin\lambda & -\cos\beta\cos\lambda & 0 \\ \sin\beta\cos\lambda & \sin\beta\sin\lambda & \cos\beta \end{Bmatrix},$$

$$\left(\frac{d\lambda}{df}\right)_{f=0.73} = 42''\sin g - 265''\sin g' - 178\sin 2\omega + 28''\sin(-2g' + 2\omega - 2\omega'),$$

$$\left(\frac{d\beta}{df}\right)_{f=0.73} = -77''\sin(\omega + \lambda_0) - 115''\sin(\omega - \lambda_0),$$

and,

$$\begin{Bmatrix} d\xi \\ d\eta \\ d\varsigma \\ dI \\ df \\ x \\ y \end{Bmatrix} \left[\sec\pi'' \begin{Bmatrix} -\sin(p-\pi'') \\ -\cos(p-\pi'') \\ 1 \end{Bmatrix} \begin{Bmatrix} A & B & C & D & E & 0 & 0 \\ A' & B' & C' & D' & E' & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{R_{\mathfrak{C}}'}{R_0} & \frac{\beta R_{\mathfrak{C}}'}{R_0} \end{Bmatrix} \right] =$$

$$= \{ S_0 - S_c \}.$$

(Yakowkin hypothesis: $dR_o = x + \beta_M y$)

2. Since an accurate value of $l_{\mathfrak{C}}$ cannot be obtained from the astronomical almanacs, it is necessary to make suitable corrections. Professor Koziel derived at Manchester the following formula for correcting the right-hand sides of the observation equations for $\Delta l_{\mathfrak{C}}$:

$$corr = -\Delta l_{\mathfrak{C}} \frac{h' \sec\pi''}{\Delta} \begin{Bmatrix} \sin(p-\pi'') \\ \cos(p-\pi'') \end{Bmatrix} \begin{Bmatrix} A_{12} & A_{22} \\ A_{13} & A_{23} \end{Bmatrix} \begin{Bmatrix} -\cos\beta\sin\lambda \\ \cos\beta\cos\lambda \end{Bmatrix}.$$

There are three right-hand sides: without limb corrections, with

Hayn and with Weimer charts. On November 14th, 1961, this new formulae was given to the programmer and new normal equations have been obtained starting from f = 0.73. It has also been checked that the mean value of the corrections $\Delta \ell_{\ell}$ for the period in question alters the d λ in the right direction and does not change other unknowns.

3. On October 18th, 1961, Mrs. Gorman received for programming the formulae obtained by Professor Koziel in Manchester for starting on the other side of the critical value of f = 0.662:

1) new formulae for dλ and dβ ; and

2) the changes of right-hand sides for f_0 = 0.60:

$$\Delta\left\{s_0 - s_c\right\} = \frac{h' \sec \pi'}{\Delta} \begin{Bmatrix} \sin(p - \pi') \\ \cos(p - \pi') \end{Bmatrix} \begin{Bmatrix} A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{Bmatrix}$$

$$\times \begin{Bmatrix} -0.0899 & -0.0553 \\ -0.9944 & +0.0050 \\ 0 & +0.9985 \end{Bmatrix} \begin{Bmatrix} d\lambda \\ d\beta \end{Bmatrix} .$$

4. Next, the coefficients of the free libration in longitude should be computed:

$$\tau = A \sin(a + a't) + \text{forced libration in longitude};$$

$$A \sin a = U$$

$$A \cos a = W$$

The following new formulae for the adjustment with free libration were given to the programmer on November 28th, 1961:

$$\left\{\begin{bmatrix} d\xi \\ d\eta \\ d\varsigma \\ dI \\ df \\ U \\ W \\ x \\ y \end{bmatrix}\right\} \left[\sec \pi'' \left\{\begin{matrix} -\sin(p-\pi'') \\ -\cos(p-\pi') \\ 1 \end{matrix}\right\} \left\{\begin{matrix} A & B & C & D & E & F & G & 0 & 0 \\ A' & B' & C' & D' & E' & F' & G' & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{R'_{\text{(}}}{R_0} & \frac{\beta R'_{\text{(}}}{R_{\text{(}}} \end{matrix}\right\} \right] = \left\{ s_c - s_c \right\}$$

$$\left\{\begin{matrix} F & G \\ F' & G' \end{matrix}\right\} = -\frac{h'}{\Delta} \left\{\begin{matrix} \cos a't \\ \sin a't \end{matrix}\right\} \left\{\begin{matrix} A_{12} & A_{22} \\ A_{13} & A_{23} \end{matrix}\right\} \left\{\begin{matrix} 0.0899 \\ 0.9944 \end{matrix}\right\}.$$

From the Kazan series first approximations of the amplitude of free libration are:

A = 19'' (without limb corrections); and

A = 8'' (with Hayn charts).

These are preliminary results.

The last part of the work concerns the formation of the coefficients of the normal equations for the solution of the problem in the second and last approximations. The respective formulae were given by K. Koziel to the programmer in December 1961 and February 1962. Instead of the unknown df, two unknowns, df_1 and df_2 have been used in order to avoid the nonlinearity of the problem, caused by the effects of a small divisor (i.e. the co-efficient to the term of argument 2ω in the forced libration in longitude). It is interesting to note that the capacity of "Mercury" was found too small for carrying out such extended calculations as the adjustment of the Bamberg series and these had therefore to be divided into sections. In each part we have six sets of systems of normal equations with 6 - 10 unknowns.

Given below are the preliminary mean errors of one observation:

	Bamberg.	Kazan.
Objective diameter	184 mm	106 mm
Focal length	2.7 m.	1.7 m.
Without limb correction	$\pm 0''.80$	$\pm 0''.87$
With Hayn chart	$\pm 0''.45$	$\pm 0''.48$
With Weimer chart	$\pm 0''.43$	$\pm 0''.52$

Professor Koziel has also prepared the programming for obtaining the s_c (divided into five parts) and a big job has been done in checking this programme. If possible, he would like to continue the libration calculations and extend them to cover the period 1841 to 1945 in order to obtain not only the classical libration constants, but also the free libration constants in inclination B, b and in node C, c.

The actual solutions of the normal equations for the definitive (second-approximation) values of the libration constants were not yet carried out before Professor Koziel and his colleagues left Manchester for Krakow by the end of the period covered by this report. As is evident from the foregoing account, the bulk of the time available to the Polish investigators at Manchester has been spent in the formulation of the coefficients of the equations of condition based on lunar heliometric observations from the years 1877 - 1915, and of the coefficients of the normal equations based upon them. This called for an amount of computation which would have been wholly impracticable without access to a modern electronic computer - such as the Ferranti "Mercury" of the University of Manchester - which are not available in Poland. On the other hand, the solution of the normal equations themselves does not call for the use of an electronic computer, and can be effectively performed with the aid of desk-type computers as are available in Krakow. The final results of such computations are awaited with interest, and should be published in the near future.

(b) Evaluation and Measurement of the Paris Lunar Negatives.

In addition to a modern re-discussion of lunar heliometric work, performed by the visual observers throughout the past century, photographic material of potentially equal value for the determination of the shape of the Moon has been known to exist at the Paris Observatory, where several thousand lunar photographs taken with the Observatory's "grand equatoreal coudé" of 60 cm aperture and 18 m focal length were accumulated between 1890 and 1910 by MM Loewy and Puiseux. The main value of this material rests on the fact that it covers in time the entire amplitude of lunar libration. On the other hand, although many photographs of this series are of excellent quality, their emulsions cannot compete with more modern photographic plates; and, above all, on account of their age, these plates cannot be transported elsewhere in bulk.

In order to evaluate their potential usefulness for the present-day selenodetic research, and to undertake such measurements of them as may be desirable, sub-contractual arrangements have been made with Dr. Th. Weimer, of the Paris Observatory (Carte-du-ciel division) staff, vice-president of the I.A.U. Commission No. 17 (Selenodesy) and a well-known expert in this field. Pursuant to these arrangements, Dr. Weimer visited the United States in the summer of 1961 for conferences with Mr. Hunt at GRD, AFCRC, and Dr. Markowitz at the U.S. Naval Observatory in Washington, concerning the instrumental equipment which he would need at Paris to embark on such measurements of the selected lunar negatives as may be desirable.

Agreement was reached, and plans drafted for the modernization of rooms and improvement of the measuring engine on loan at Paris from the U.S. Naval Observatory, to be done partly at Paris and partly in the United States. The terms of a sub-contract to finance these developments from the funds available to us have been worked out, and signed on March 7th 1961, by Professor Danjon, Director of the Paris Observatory, and by Professor Kopal on behalf of the University of Manchester. In the meantime, Dr. Weimer embarked on a selection of the Paris negatives, taken at different angles of libration, on which craters with dimensions between 2 - 5 kms are measurable. A list of such objects suitable for measurement was published by Dr. Weimer in an Appendix to the Proceedings of the <u>First Conference on Lunar Topography</u>, held at Bagnères de Bigorre in April 1960.

(c) <u>Development of Methods for determination of the relative heights of shallow surface features.</u>

The aim of the research carried out under this heading by Messrs T. W. Rackham and E. D. Dale, working under the supervision of Professor Kopal at the University of Manchester, has been to study the properties of lunar micro-relief in the region of the maria by a method based on micro-photometric tracings of the Pic-du-Midi lunar negatives. This micro-relief, small as it is when compared with the topography of the craters of mountain chains, attains some prominence on the otherwise flat maria. In general, therefore, the study is concerned with the relief of the maria, and hopes to contribute towards a clearer understanding of the nature of their surface.

Topographic features amounting to no more than 20 metres in height, and inclined at an angle of the order of one degree, can be measured significantly by a comparison of the photographic densities of the negative on a sloping and level region. By a comparison of a density of the image of a ridge (or dome) with equal density on the mare at a spot where the Sun stands higher (or lower) above the horizon, actual slopes over the ridge (or dome) can be established. Graphical integration then furnishes a series of spot heights over the particular uneveness of the surface; and by using a raster technique a contour map of the formation can be drawn.

Mr. Dale applied this technique in some detail to the Serpentine ridge in Mare Serenitatis, as well as to a representative lunar dome in the Arago region. The quantitative results of his work are contained in an M.Sc. thesis, now in preparation, to be submitted to the University of Manchester in the fall of 1962; and will also be published as a Technical Scientific Note under this contract.